International Journal of Electrical, Electronics and Computer Engineering **1**(2): 15-21(2012)
ISSN No. (Online) : 2277-2626
Special Edition for Best Papers of Michael Faraday IET India Summit-2012, MFIIS-12

Control Oriented LFT Modeling of a Non Linear MIMO system

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(Received 15 October, 2012 Accepted 01 December, 2012)

ABSTRACT: This paper presents a procedure for parametric uncertainty modeling of a highly nonlinear *multi- input multi- output* (MIMO) cross coupled system for the purpose of Linear Fractional Transformation (LFT) model realization. A novel technique has been considered to represent the control oriented LFT modeling of linear parametric models. A systematic approach is to find the equivalent representations of manipulation of rationally dependent parametric matrices. Model uncertainty arises when the system gain and the parameters are not precisely known, or may vary over a given range. One may also have unstructured uncertainties, by which one means complex parameter variations satisfying given magnitude bounds. Linear Fractional Transformation is objects of study for robust and Linear Parameter Varying (LPV) control. For robust control LFT model is desirable. A nonlinear strongly coupled dynamics laboratory twin rotor MIMO system, which constitutes challenges for many classical linear control techniques, has been consider as a candidate system. A two-degree-of-freedom (2DOF) design framework has been adopted for the formulation of the LFT modeling.

Index Terms—Dynamic perturbation, LFT modeling, MIMO system, unstructured uncertainty.

I. INTRODUCTION

In the field of control theory linear multivariable systems is a mature subject with a variety of successful application.

Although the nonlinear control theory is quite attractive and so many researchers have recently showed an active interest in the development and application of nonlinear control methodologies. It is very difficult to design such controller for high level nonlinear cross coupled multivariable dynamic system. Robust control is inherently about model uncertainty, particularly focusing on the implications of model uncertainty for decisions and the model of the system has been represented in the form of LFT. This paper concerns with the systemic control oriented modeling of uncertain non-linear system whose model vary due to changes in the system configuration and operating conditions. The differences between the linearized mathematical model and the actual system, presence of disturbance signal, and the model order reduction have been considered for the model uncertainty. A highly nonlinear strongly coupled laboratory scaled twin rotor *multi-input multi-output* system has been considered as a candidate system.

The purpose of the paper is to develop a system model to characterize system variations as uncertainty. Model uncertainty generally has nominal system model and the unknown transfer function matrix. This modeling approach leads to the system in the form of linear fractional transformation (LFT). A general descriptor type LFT representation of rational parametric matrices is a generalized represented of arbitrary rationally dependent multivariate functions in LFT-form [1]. A technique has been proposed to model uncertain nonlinear systems whose models vary due to changes in the system configuration and operating conditions [2]. A unified frame work for parameter estimation problems which arise in a system identification context. The parameters to be estimated appear in a LFT with a known constant matrix M [3]. A general procedure is to approximate a parametric linear fractional representation (LFR) with a reduced order LFR [4]. Earlier studies present a systematic approach for the generation of uncertainty models described by LFT and report on recently developed symbolic and numerical software to assist the generation of low order LFT-based uncertainty models. It contributes a systematic approach to represent a nonlinear cross couple dynamic systems to a matrix polynomial of any order and any number of parameters as an LFT [5].

Physical systems are generally multi input multi output system and maximum systems are non-linear. In non linear MIMO system complexity is increased by the increase of input output. Helicopter is an aircraft which is lifted and propelled by one or more horizontal rotors consisting of two or more rotor blades. It can take off and land vertically and to maintain a steady hover in the air over a single point on the ground. Two degree of freedom helicopter equipment (Twin Rotor MIMO System developed by Feedback) available in the laboratories of the Advanced Control Systems Research. This model is a good multivariable control benchmark widely used in the literature. It allows illustrating the control of helicopter mechanics with two degrees of freedom that rotates around two directions.

In mathematical modeling of TRMS, the system is modeled in terms of vertical one-degree-of-freedom (1DOF), horizontal (1DOF), and two-degree-of-freedom (2DOF) dynamics using Newtonian as well as Lagrangian methods [6]. Stable adaptive model for predictive control approach has been proposed for constrained nonlinear systems. This method is known as multistep Newton-type control strategies however, the formulation here differs from the original one [7]. A mathematical model of Twin Rotor Aero dynamical System containing experimental characteristics has been used to design the controllers for tracking [8]. Genetic modeling and vibration control of twin rotor MIMO system has introduced global search technique of GA is used to identify the parameters of the TRMS based on one-step-ahead prediction [9]. Recent study shows unknown nonlinearities of TRMS has been estimated by neural network whose weights are adaptively adjusted [10].

All the linear and non linear modeling of the TRMS has some approximation is used in order to get some necessary knowledge. Due to this approximation some uncertainty is introduced into the system and which is unavoidable in a real control system. All this uncertainty is lumped into a single block. Uncertainty block affects the input – output relationship of the LFT. This LFT modeling is essential for the robust Control.

II. MODEL UNCERTAINTY

Most control designs are based on the use of the mathematical modeling of the real systems. A mathematical model produces a map from inputs to responses. The quality of the model depends on how closely its response matches to the real system. Since no single fixed model can respond exactly like true plant. This discrepancy introduce due to the unmodelled dynamics, neglecting the nonlinearities in the modeling, effects of reduced order model and system parameter variation due to environmental changes. This discrepancy between the derived model and the actual plant has been considered as model uncertainty in robust control theory [11]. Stability and performance of the control system is very much influenced by the model uncertainty.

The model uncertainty is classified in two categories, disturbance signals and dynamic perturbations. Disturbance signals occur due to the input output disturbance, sensor noise actuator noise, etc. Dynamic perturbations represent the discrepancy between mathematical model and the actual dynamics of the system

Many dynamic perturbations that may occur in different parts of a system can, however, be lumped into one single perturbation block Δ , for instance, some unmodelled, high-frequency dynamics. This uncertainty representation is referred to as "unstructured" uncertainty.In the case of linear, time-invariant systems, the block *Δ* may be represented by an unknown transfer function matrix.

There are many different types of uncertain system model representation [12] and the form of model to be used depends on type of uncertainty expected and the tractability of robust control problem corresponding to this uncertain system model.

Generation of LFT Models

Linear fractional Transformation (LFT) plays an important role in modeling parametric uncertainty in linear systems. LFT based model representation [11] are very useful to model real parametric uncertainty entering rationally in the system matrix. These models are used in the robust control application like H_∞ control or μ control.

Consider a complex matrix M as

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \in C^{(p_1+p_2)\times(q_1+q_2)} \qquad (1)$$

and let $\Delta_l \in C^{q_2\times p_2}$ and $\Delta_u \in C^{q_1\times p_1}$ be two other complex matrices. The lower LFT with respect to Δ_l as

$$F_l(M,\bullet): C^{q_2\times p_2} \to C^{p_1\times q_1}$$

With mathematical expression

$$F_l(M,\Delta_l) := M_{11} + M_{12}\Delta_l(I - M_{22}\Delta_l)^{-1}M_{21} \qquad (2)$$

provided that the inverse $(I - M_{22}\Delta_l)^{-1}$ is exists. The upper LFT with respect to Δ_u as

$$F_u(M,\bullet): C^{q_1\times p_1} \to C^{p_2\times q_2}$$

With

$$F_u(M,\Delta_u) := M_{22} + M_{21}\Delta_u(I - M_{11}\Delta_u)^{-1}M_{12} \qquad (3)$$

also provided that the inverse $(I - M_{11}\Delta_u)^{-1}$ is exists. Matrix M is known as the co- efficient matrix. The lower and upper LFT is shown in the figure.

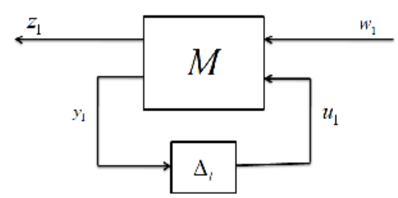

Fig. 1. Lower LFT Representations.

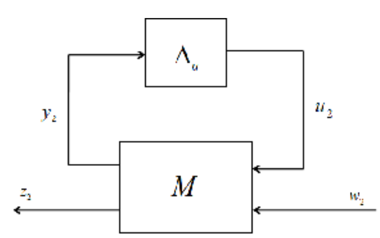

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Fig. 2. Upper LFT Representations.

The lower LFT representation express by the following set of equations:

$$\begin{bmatrix} z_1 \\ y_1 \end{bmatrix} = M \begin{bmatrix} w_1 \\ u_1 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} w_1 \\ u_1 \end{bmatrix}, \tag{4}$$

$$u_1 = \Delta_l y_1 \tag{5}$$

And the upper LFT representation express by

$$\begin{bmatrix} y_2 \\ z_2 \end{bmatrix} = M \begin{bmatrix} u_2 \\ w_2 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} u_2 \\ w_2 \end{bmatrix}, \tag{6}$$

$$u_2 = \Delta_u y_2 \tag{7}$$

III. Twin Rotor MIMO System

Among autonomous flying systems, helicopters have particularly dynamic features. The main difficulties in designing controllers for them follow from nonlinearities and cross couplings. A 2-DOF model is considered, and unlike in most of the recent works, aero dynamical forces and torques are introduced into the modeling of the system. The system is interesting because it makes it possible to perform various experiments in the field of modeling, identification and control theory.

Figure 3 shows a laboratory model of the Twin rotor MIMO systems (TRMS), are basically a multi inputs multi outputs system (MIMO). In certain aspects its behavior resembles that of a helicopter. It consists of a beam pivoted on its base in such a way that it can move freely both in the horizontal and vertical plane. At the both end of the beam, there are two propellers driven by the two dc motors. The TRMS system has a main and tail rotor for generating vertical and horizontal thrust. Main rotor produces a thrust to lift the beam in vertical plane and tail rotor produce a force to make the beam turn left or right in a horizontal plane. The TRMS system has two degree of freedom (2DOF) movement, one degree of freedom (1 DOF) in the vertical plane and one degree of freedom (1 DOF) in the horizontal plane movement. The system is equipped with a counterweight hanging from the beam, which is used for balancing the angular momentum in steady state or with load.

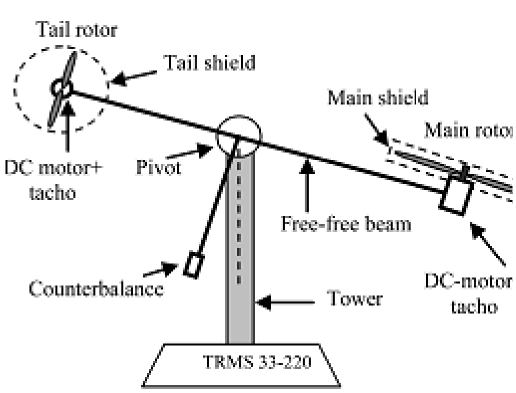

Fig. 3. Twin Rotor MIMO system.

The aerodynamics forces of TRMS are controlled by varying the speed of the motors. Therefore, supply voltages of the DC motors are the control input of the system. It is highly non linear system and process strong cross coupling between the pitch and yaw axis. In this study Lagrangian mathematical model is taken in to the consideration.

IV. LFT Modeling of TRMS

Mathematical modeling of TRMS, the system is modeled in terms of vertical one-degree-of-freedom (1DOF), horizontal (1DOF), and two-degree-of-freedom (2DOF) dynamics using Newtonian as well as Lagrangian methods [6]. Lagrangian based mathematical model is used for the linear fractional transformation (LFT) modeling. Different types of sensors are used in the actual system so disturbance signals introduce into the system. The difference between the mathematical model and the actual model is due to the presence of the dynamical perturbation and linearization of the non-linearity. This uncertainty can be lumped into a block Δ.

$$\begin{bmatrix} (J_1 cos^2(\alpha_v) + J_2 sin^2(\alpha_v) + h^2 m_{T_1} + h^2 m_{T_2} + J_3 & m_{T_1} l_{T1} h \sin(\alpha_v) - m_{T_2} l_{T_2} h \cos(\alpha_v) \\ m_{T_1} l_{T1} h \sin(\alpha_v) - m_{T_2} l_{T_2} h \cos(\alpha_v) & J_1 + J_2 \end{bmatrix} \begin{bmatrix} \ddot{\alpha}_h \\ \ddot{\alpha}_v \end{bmatrix}$$
$$+ \begin{bmatrix} k_h & 0 \\ 0 & k_v \end{bmatrix} \begin{bmatrix} \dot{\alpha}_h \\ \dot{\alpha}_v \end{bmatrix} + \begin{bmatrix} [m_{T_1} l_{T_1} h \cos(\alpha_v) + m_{T_2} l_{T_2} h \sin(\alpha_v)] \dot{\alpha}_v^{\,2} + [2(J_2 - J_1) \sin(\alpha_v) \cos(\alpha_v)] \dot{\alpha}_h \dot{\alpha}_v \\ [J_1 - J_2] \sin(\alpha_v) \cos(\alpha_v) \dot{\alpha}_h^{\,2} + g[m_{T_1} l_{T_1} \cos(\alpha_v) + m_{T_2} l_{T_2} \sin(\alpha_v)] \end{bmatrix}$$
$$+ \begin{bmatrix} 0 & -k_m \cos(\alpha_v) \\ -k_t & 0 \end{bmatrix} \begin{bmatrix} \dot{\omega}_h \\ \dot{\omega}_v \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} M_{prop,h} \\ M_{prop,v} \end{bmatrix} \tag{8}$$

$$D(\alpha_h, \alpha_v) \begin{bmatrix} \ddot{\alpha}_h \\ \ddot{\alpha}_v \end{bmatrix} + C(\alpha_{h,} \alpha_v, \ddot{\alpha}_h, \ddot{\alpha}_v) + g(\alpha_h, \alpha_v) = \begin{bmatrix} M_{prop,h} \\ M_{prop,v} \end{bmatrix} \tag{9}$$

After linearization of the (8) under the assumptions of a small deviation of the horizontal and vertical position, one obtain the following equation

$$\begin{bmatrix} (J_1 + h^2 m_{T_1} + h^2 m_{T_2} + J_3) & -m_{T_2} l_{T_2} h \\ -m_{T_2} l_{T_2} h & J_1 + J_2 \end{bmatrix} \begin{bmatrix} \ddot{\alpha}_h \\ \ddot{\alpha}_v \end{bmatrix} + \begin{bmatrix} k_h & 0 \\ 0 & k_v \end{bmatrix} \begin{bmatrix} \dot{\alpha}_h \\ \dot{\alpha}_v \end{bmatrix} \tag{10}$$
$$+ \begin{bmatrix} 0 & 0 \\ 0 & g m_{T_2} l_{T_2} \end{bmatrix} \begin{bmatrix} \alpha_h \\ \alpha_v \end{bmatrix} + \begin{bmatrix} 0 & -k_m \\ -k_t & 0 \end{bmatrix} \begin{bmatrix} \dot{\omega}_h \\ \dot{\omega}_v \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} M_{prop,h} \\ M_{prop,v} \end{bmatrix}$$

$$D\ddot{\alpha} + C\dot{\alpha} + E\alpha + Kd = Tp \tag{11}$$

$$\ddot{\alpha} = -D^{-1}C\dot{\alpha} - D^{-1}E\alpha - D^{-1}Kd + D^{-1}Tp \tag{12}$$

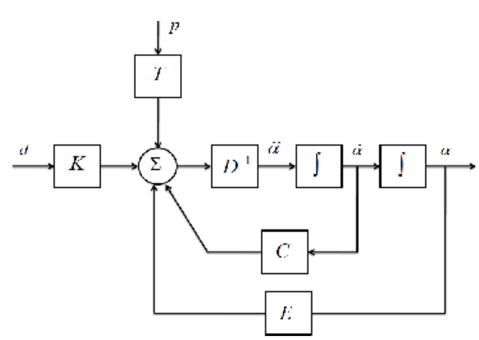

Fig. 4. Block diagram of Twin rotor MIMO system.

Based on the practical considerations, the variations of the moment of inertia of the free-free beam J_1, moment of inertia of the counterbalance beam J_2 and viscous friction co-efficient k_1 and k_2 are considered. It is assumed that the moments of inertia are constants but with possible relative error of 30% around the nominal values; similarly, the friction coefficients may have with 15% relative errors [13].
Therefore, the actual moments of inertia are presented as

$$J_i = \bar{J}_i(1+p_i\delta_{Ji}), i=1,2 \quad (13)$$

Where $\bar{J}_i$ is the nominal value of the corresponding moment of inertia, $p_i = 0.3$ is the maximum relative uncertainty in each of these moments and $-1 \le \delta_{Ji} \le 1, i=1,2$.

$$D = \bar{D} + D_p\Delta_J \quad (14)$$

where the elements $\bar{D}$ are determine by the nominal values of the moment of inertia.

$$\bar{D}=\begin{bmatrix}\bar{J}_1+h^2\bar{m}_{T_1}+h^2\bar{m}_{T_2}+\bar{J}_3 & -\bar{m}_{T_2}\bar{l}_{T_2}\bar{h}\\ -\bar{m}_{T_2}\bar{l}_{T_2}\bar{h} & \bar{J}_1+\bar{J}_2\end{bmatrix}, D_p=\begin{bmatrix}\bar{J}_{1p_1} & 0\\ 0 & \bar{J}_{2p_2}\end{bmatrix} and \Delta_J=\begin{bmatrix}\delta_{J_1} & 0\\ 0 & \delta_{J_2}\end{bmatrix}$$

The matrix D^{-1} can be represented as an upper linear fractional transformation (LFT).

$$D^{-1} = F_U(Q_J,\Delta_J) = Q_{J_{22}} + Q_{J_{21}}\Delta_J(I_2 - Q_{J_{11}}\Delta_J)^{-1}Q_{J_{12}} \quad (15)$$

$$Q_{J_{11}} = -\bar{D}^{-1}D_p, Q_{J_{12}} = \bar{D}^{-1}, Q_{J_{21}} = -\bar{D}^{-1}D_p and Q_{J_{22}} = \bar{D}^{-1}$$

$$Q_J = \begin{bmatrix}Q_{J_{11}} & Q_{J_{12}}\\ Q_{J_{21}} & Q_{J_{22}}\end{bmatrix} = \begin{bmatrix}-\bar{D}^{-1}D_p & \bar{D}^{-1}\\ -\bar{D}^{-1}D_p & \bar{D}^{-1}\end{bmatrix} \quad (16)$$

Table -1

Nominal values of the parameters

Symbol(Unit)	value	Symbol(Unit)	value
$m_t(kg)$	0.015	$m_h(kg)$	0.014
$m_{tr}(kg)$	0.221	$l_t(m)$	0.282
$m_{ts}(kg)$	0.119	$l_m(m)$	0.246
$m_m(kg)$	0.014	$l_b(m)$	0.290
$m_{mr}(kg)$	0.236	$l_{cb}(m)$	0.276
$m_{ms}(kg)$	0.219	$r_{ms}(m)$	0.155
$m_b(kg)$	0.022	$r_{ts}(m)$	0.100
$m_{cb}(kg)$	0.068	$h(m)$	0.240

Nominal values of the TRMS are taken from the Feedback instrument manuals [14].
Let us now consider the uncertainties of the friction co-efficient. Here k_1 and k_2 are viscous friction coefficients in horizontal and vertical position of the TRMS.

Therefore, the actual viscous friction coefficient are presented as

$$k_i = \bar{k}_i(1+s_i\delta_{k_i}), i=1,2 \quad (17)$$

where $\bar{k}_i$ is the nominal value of the corresponding viscous friction, $s_i = 0.15$ is the maximum relative uncertainty in each of these coefficient and $-1 \le \delta_{k_i} \le 1, i=1,2$.

$$C = \bar{C} + C_s\Delta_k \quad (18)$$

where the elements $\bar{C}$ are determine by the nominal values of the viscous friction coefficient.

$$\bar{C}=\begin{bmatrix}\bar{k}_1 & 0\\ 0 & \bar{k}_2\end{bmatrix}, C_s=\begin{bmatrix}\bar{k}_1s_1 & 0\\ 0 & \bar{k}_2s_2\end{bmatrix} and \Delta_k=\begin{bmatrix}\delta_{k_1} & 0\\ 0 & \delta_{k_2}\end{bmatrix}$$

The matrix C^{-1} can be represented as an upper linear fractional transformation (LFT).

$$C^{-1} = F_U(Q_k,\Delta_k) = Q_{k_{22}} + Q_{k_{21}}\Delta_k(I_2 - Q_{k_{11}}\Delta_k)^{-1}Q_{k_{12}} \quad (17)$$

$$Q_{k_{11}} = -\bar{C}^{-1}C_s, Q_{k_{12}} = \bar{C}^{-1}, Q_{k_{21}} = -\bar{C}^{-1}C_s and Q_{k_{22}} = \bar{C}^{-1}$$

$$Q_k = \begin{bmatrix}Q_{k_{11}} & Q_{k_{12}}\\ Q_{k_{21}} & Q_{k_{22}}\end{bmatrix} = \begin{bmatrix}-\bar{C}^{-1}C_s & \bar{C}^{-1}\\ -\bar{C}^{-1}C_s & \bar{C}^{-1}\end{bmatrix} \quad (18)$$

To represent the TRMS model as a LFT of the real uncertain parameters δ_{J_1}, δ_{J_2} and δ_{k_1}, δ_{k_2}. We first extract out the uncertain parameters and then denote the inputs and outputs of Δ_J and Δ_K as y_J, y_K and u_J, u_K respectively.

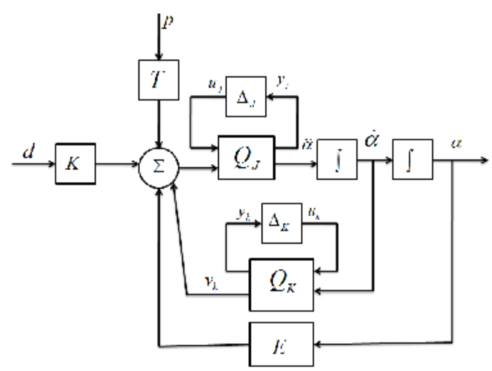

Fig.5. System block diagram with uncertain parameter

$$\begin{bmatrix}y_i\\ \ddot{\alpha}\end{bmatrix} = \begin{bmatrix}-\bar{D}^{-1}D_p & \bar{D}^{-1}\\ -\bar{D}^{-1}D_p & \bar{D}^{-1}\end{bmatrix}\begin{bmatrix}u_j\\ Tp-(Kd+v_k)\end{bmatrix} \quad (19a)$$

$$\begin{bmatrix}y_k\\ v_k\end{bmatrix} = \begin{bmatrix}-\bar{C}^{-1}C_s & \bar{C}^{-1}\\ -\bar{C}^{-1}C_s & \bar{C}^{-1}\end{bmatrix}\begin{bmatrix}u_k\\ \dot{\alpha}\end{bmatrix} \quad (19b)$$

$$u_j = \Delta_j y_j \quad (20a)$$

$$u_k = \Delta_k y_k \tag{20b}$$

The TRMS state vector $x = [x_1 \quad x_2 \quad x_3 \quad x_4]^T$ is defined by $x_1 = \alpha_h, x_2 = \alpha_v, x_3 = \dot{\alpha}_h, x_4 = \dot{\alpha}_v$.

Hence $\dot{\alpha} = [\dot{x}_1 \quad \dot{x}_2]^T, \ddot{\alpha} = [\dot{x}_3 \quad \dot{x}_4]^T$

$$\text{Output } y_{p_1} = k_h \alpha_h \tag{21a}$$

$$y_{p_2} = k_v \alpha_v \tag{21b}$$

Now introducing the output vector $y_p = [y_{p_1} \quad y_{p_2}]^T$

So $$y_p = E_p \alpha \tag{22}$$

Where $E_p = \begin{bmatrix} k_h & 0 \\ 0 & k_v \end{bmatrix}$

From the state equations of the TRMS the input output relationship is summarized as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ y_j \\ y_k \\ y_p \\ y \end{bmatrix} = \Pi \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ u_j \\ u_k \\ p \\ d \end{bmatrix} \tag{23}$$

$$\Pi = \left[\begin{array}{cc:cc:cc} 0_{2\times2} & I_{2\times2} & 0_{2\times2} & 0_{2\times2} & 0_{2\times2} & 0_{2\times2} \\ -\bar{D}^{-1}E & -\bar{D}^{-1}\bar{C}^{-1} & -\bar{D}^{-1}D_p & \bar{D}^{-1}\bar{C}^{-1}C_s & \bar{D}^{-1}T & -\bar{D}^{-1}K \\ \hdashline -\bar{D}^{-1}E & -\bar{D}^{-1}\bar{C}^{-1} & -\bar{D}^{-1}D_p & \bar{D}^{-1}\bar{C}^{-1}C_s & \bar{D}^{-1}T & -\bar{D}^{-1}K \\ 0_{2\times2} & \bar{C}^{-1} & 0_{2\times2} & -\bar{C}^{-1}C_s & 0_{2\times2} & 0_{2\times2} \\ \hdashline E_p & 0_{2\times2} & 0_{2\times2} & 0_{2\times2} & 0_{2\times2} & 0_{2\times2} \\ I_{2\times2} & 0_{2\times2} & 0_{2\times2} & 0_{2\times2} & 0_{2\times2} & 0_{2\times2} \end{array}\right]$$

$$\begin{bmatrix} u_j \\ u_k \end{bmatrix} = \begin{bmatrix} \Delta_j & 0 \\ 0 & \Delta_k \end{bmatrix} \begin{bmatrix} y_j \\ y_k \end{bmatrix} \tag{24}$$

The open loop model of the TRMS has four inputs (u_j, u_k, p, d) and four outputs (y_i, y_k, y_p, y).

$$\begin{bmatrix} y_j \\ y_k \\ y_p \\ y \end{bmatrix} = G_{trms} \begin{bmatrix} u_j \\ u_k \\ p \\ d \end{bmatrix} \tag{25}$$

The state space representation of the TRMS is

$$G_{trms} = \left[\begin{array}{c:cc} A & B_1 & B_2 \\ \hdashline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right]$$

The input output relation of the perturbed TRMS is described by the upper LFT

$$\begin{bmatrix} y_p \\ y \end{bmatrix} = F_U(G_{trms}, \Delta_{trms}) \begin{bmatrix} p \\ d \end{bmatrix} \tag{26}$$

with the diagonal, uncertain matrix $\Delta_{trms} = \begin{bmatrix} \Delta_J & 0 \\ 0 & \Delta_k \end{bmatrix}$ (27)

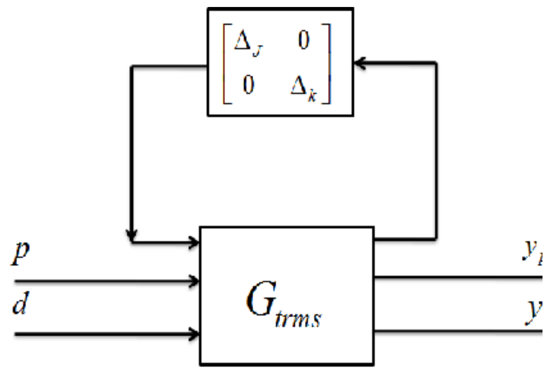

Fig.6. LFT representation of perturbed Two input two output systems

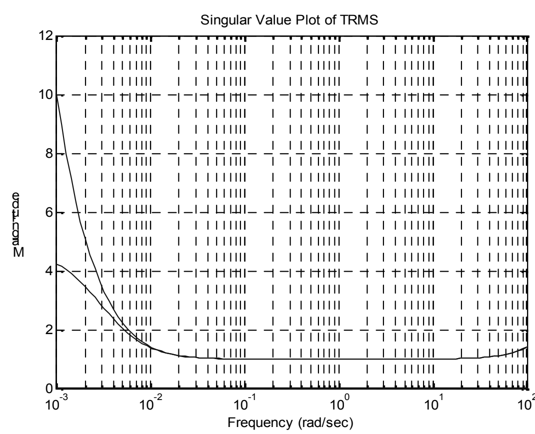

Fig: 7 Singular values of the Twin rotor MIMO system

Now consider the model of motors of the twin rotor MIMO system. From the Feedback instrument manuals consider the linear transfer function of the tail and main motors are

$$G_{tr} = \frac{K_{tr}}{T_{tr}s+1} = \frac{1}{0.3842s+1} \tag{28a}$$

$$G_{mr} = \frac{K_{mr}}{T_{mr}s+1} = \frac{1}{1.432s+1} \tag{28b}$$

It is assumed that the actual gain coefficient K_{tr} and K_{mr} are constant with relative error 10% around their nominal values and the time constants T_{tr} and T_{mr} with relative error 20% [13].

The uncertain transfer function the tail and main motor

$$G_{tr} = (1 + w_{tr}\delta_{tr})\overline{G}_{tr} \tag{29a}$$

$$G_{mr} = (1 + w_{mr}\delta_{mr})\overline{G}_{mr} \tag{29b}$$

Where $|\delta_{tr}| \le 1, |\delta_{mr}| \le 1$ W_{tr} and W_{mr} are uncertainty weights. Transfer function of the weights W_{tr} and W_{mr} are

$$W_{tr} = \frac{10s + 4.2}{s + 89.6} \qquad (30a)$$

$$W_{mr} = \frac{10s + 4}{s + 99.6} \qquad (30b)$$

Introducing the input vector $u = [u_1 \quad u_2]^T$ than $p = \overline{G}_r (I_2 + w_r \Delta_r) u$.

Where $\overline{G}_r = \begin{bmatrix} \overline{G}_{tr} & 0 \\ 0 & \overline{G} \end{bmatrix}, w_r = \begin{bmatrix} w_{tr} & 0 \\ 0 & w_{mr} \end{bmatrix}$ and $\Delta_r = \begin{bmatrix} \delta_{tr} & 0 \\ 0 & \delta_{mr} \end{bmatrix}$

Let the input and output of the uncertainty block denoted by u_{tr}, u_{mr} and y_{tr}, y_{mr} respectively.

$$u_r = [u_{tr} \quad u_{mr}]^T, y_r = [y_{tr} \quad y_{mr}]^T$$

Frequency response of the main motor and the tail motor of the TRMS is given in figure 8 and figure 9 respectively.

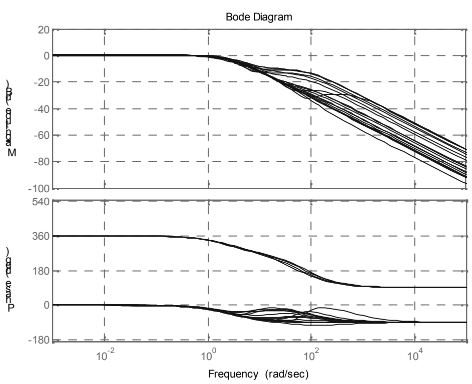

Fig. 8. Frequency response of the main motor.

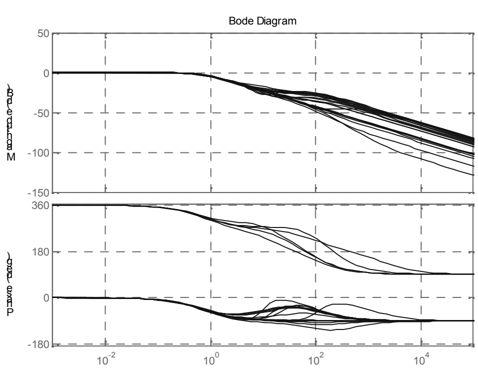

Fig. 9. Frequency response of the tail motor.

Now the motor model becomes $\begin{bmatrix} y_r \\ p \end{bmatrix} = G_m \begin{bmatrix} u_r \\ u \end{bmatrix}$ (31)

$$u_r = \Delta_r y_r \qquad \ldots(32)$$

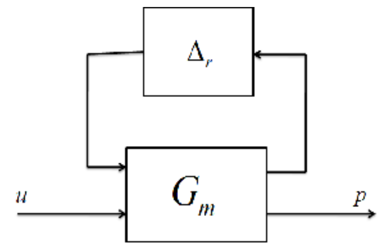

Fig. 10. LFT representation of the perturbed.

Output of the perturbed motor is supplying the propulsive force to the perturbed TRMS. Note that the Δ_r is a complex uncertainty, while Δ_J *and* Δ_K are real uncertainties.

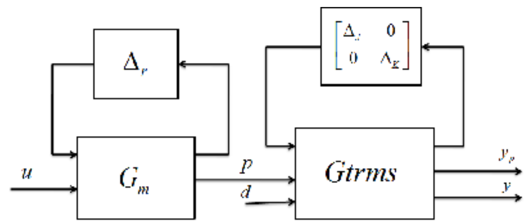

Fig.11. Uncertainty modeling of Twin Rotor MIMO system with motors (main and tail motor).

The model of the total Twin Rotor MIMO system is given by the equations given below

$$\begin{bmatrix} y_r \\ y_j \\ y_k \\ y_p \\ y \end{bmatrix} = G_{sys} \begin{bmatrix} u_r \\ u_j \\ u_k \\ u \\ d \end{bmatrix} \qquad (33)$$

Where G_{sys} is determined by the matrices G_m and G_{trms}.

$$\begin{bmatrix} y_p \\ y \end{bmatrix} = F_U(G_{sys}, \Delta) \begin{bmatrix} u \\ d \end{bmatrix} \qquad (34)$$

With the diagonal matrix $\Delta = \begin{bmatrix} \Delta_r & 0 & 0 \\ 0 & \Delta_j & 0 \\ 0 & 0 & \Delta_k \end{bmatrix}$ (35)

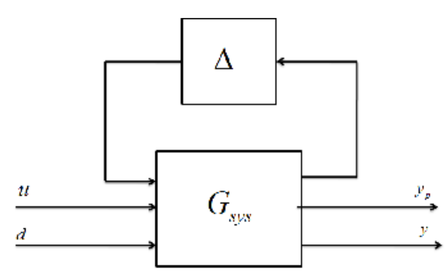

Fig. 12. LFT representation of TRMS system with uncertainty.

V. DISCUSSION

This paper contributes an excellent methodology to represent the model uncertainties due to the difference between the linearized mathematical model and the actual system.

In modeling parametric uncertainties in linear systems the *linear fractional transformation* (LFT) plays an important role. LFT-based representations are useful to model real parametric uncertainties entering rationally in the system matrices. These models are ready to be used in robust control tools.

The laboratory model of the Twin rotor MIMO systems (TRMS), are basically a multi inputs multi outputs system (MIMO) and in certain aspects its behavior resembles that of a helicopter.

The practical helicopter has three degree of freedom (DOF) movement but the TRMS system has two degree of freedom (2DOF) movement, one degree of freedom (1 DOF) in the vertical plane and one degree of freedom (1 DOF) in the horizontal plane movement.

The input output relation of the perturbed TRMS is described by the upper LFT. In LFT modeling of system, all the unstructured uncertainty considers in a single block Δ which may be represented by an unknown transfer function matrix and is known as an uncertainty matrix. The LFT model derived using this approach has the advantage of being physically meaningful and is able to accurately represent the uncertain nonlinear model. This LFT modeling is adopted in the robust control study for uncertainty modeling. LFT modeling of TRMS is essential for the H_∞ controller design.

The results obtain by linearized model are valid only for sufficient small-angle movement in the horizontal and the vertical plane.

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